Chapter 3

Perpendicular and Parallel Lines

Section 2
# Proof and Perpendicular Lines

GOAL 1: Comparing Types of Proofs

There is more than one way to write a proof. The two-column proof on the following slide is from Lesson 2.6. It can also be written as a paragraph proof or as a flow proof. A flow proof uses arrows to show the flow of the logical argument. Each reason in a flow proof is written below the statement it justifies.

Example 1: Comparing Types of Proof

GIVEN ▸ $\angle 5$ and $\angle 6$ are a linear pair.
$\angle 6$ and $\angle 7$ are a linear pair.

PROVE ▸ $\angle 5 \cong \angle 7$
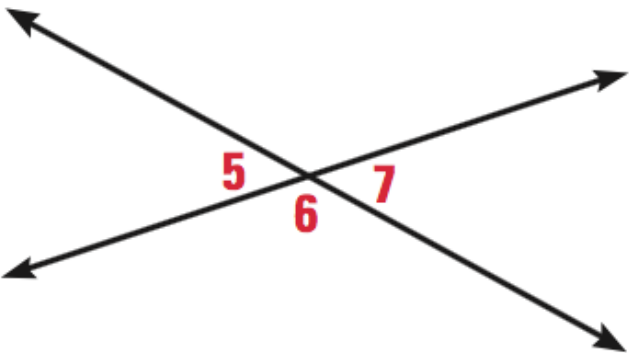
Method 1 *Two-column Proof*

Statements	Reasons
1. $\angle 5$ and $\angle 6$ are a linear pair. $\angle 6$ and $\angle 7$ are a linear pair.	1. Given
2. $\angle 5$ and $\angle 6$ are supplementary. $\angle 6$ and $\angle 7$ are supplementary.	2. Linear Pair Postulate
3. $\angle 5 \cong \angle 7$	3. Congruent Supplements Theorem

Method 2 *Paragraph Proof*

Because ∠5 and ∠6 are a linear pair, the Linear Pair Postulate says that ∠5 and ∠6 are supplementary. The same reasoning shows that ∠6 and ∠7 are supplementary. Because ∠5 and ∠7 are both supplementary to ∠6, the Congruent Supplements Theorem says that ∠5 ≅ ∠7.

Method 3 *Flow Proof*

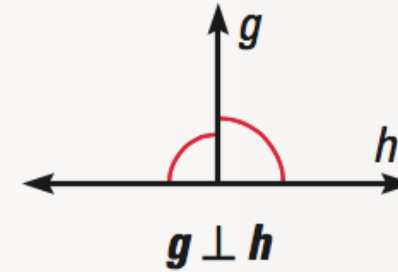| ∠5 and ∠6 are a linear pair.	→	∠5 and ∠6 are supplementary.	
Given		Linear Pair Postulate	∠5 ≅ ∠7
∠6 and ∠7 are a linear pair.	→	∠6 and ∠7 are supplementary.	Congruent Supplements Theorem
Given		Linear Pair Postulate	
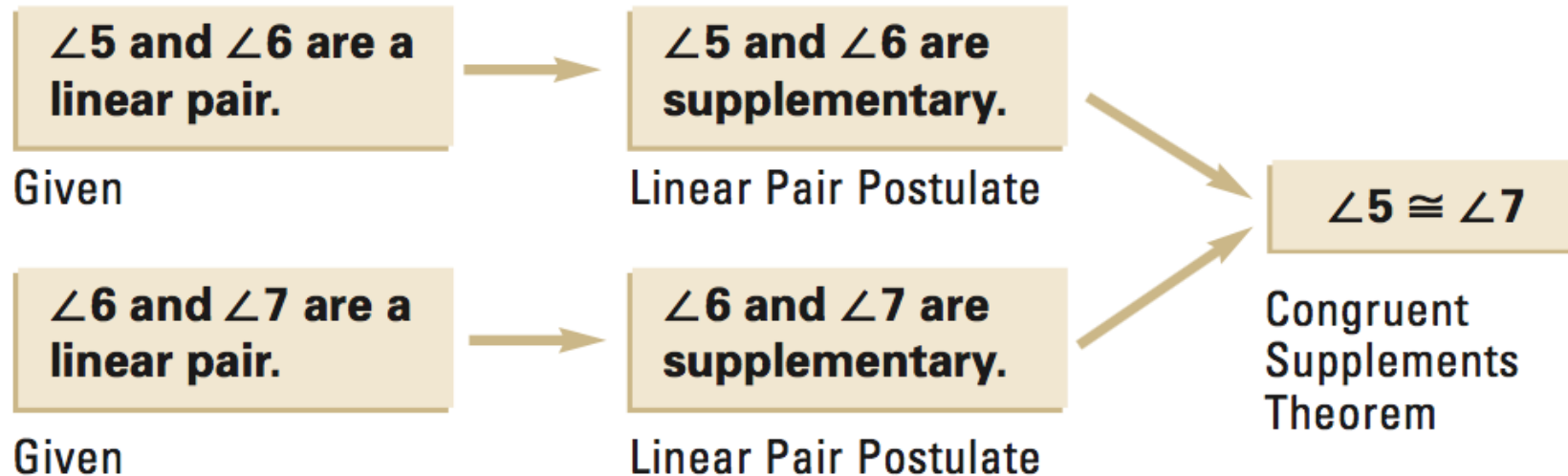
GOAL 2: Proving Results About Perpendicular Lines
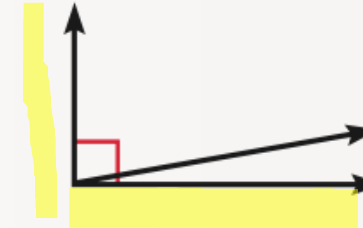
THEOREMS

THEOREM 3.1

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

$g \perp h$

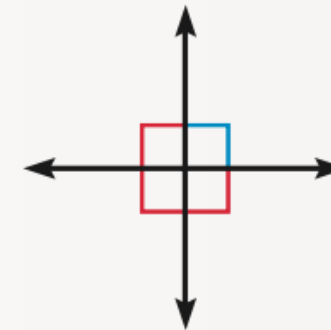**THEOREM 3.2**

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

THEOREM 3.3

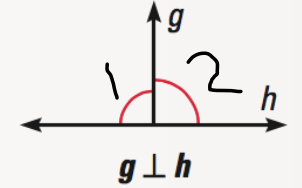If two lines are perpendicular, then they intersect to form four right angles.

We will prove Theorem 3.2 and 3.3 in Exercises 17-19.

Example 2: Proof of Theorem 3.1

THEOREM 3.1 Given

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular. Prove

$g \perp h$

Write a proof of Theorem 3.1.

Statements	Reasons
1 - <1 & <2 are a linear pair	Given
<1 cong. <2	
2 - <1 & <2 are supplementary	Linear Pair Postulate
3 - m<1 + m<2 = 180	Def. of supplementary <s
4 - m<1 = m<2	Def. of congruent <s
5 - m<1 + m<1 = 180	Substitution
6 - 2(m<1) = 180	Simplify/C.L.T. (Distributive)
7 - m<1 = 90	Division
8 - <1 is a right angle	Def. of right <s
9 - g is perp. to h	Def. of perp. lines

$x + x = 180$

$2x = 180$

$x = 90$

TYPES OF PROOFS

You have now studied three types of proofs.

1. **TWO-COLUMN PROOF** This is the most formal type of proof. It lists numbered statements in the left column and a reason for each statement in the right column.

2. **PARAGRAPH PROOF** This type of proof describes the logical argument with sentences. It is more conversational than a two-column proof.

3. **FLOW PROOF** This type of proof uses the same statements and reasons as a two-column proof, but the logical flow connecting the statements is indicated by arrows.